ABSTRACT

Optimization of Insertion Cost for Transfer Trajectories to Libration Point Orbits

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Introduction

The objective of this work is the development of efficient techniques to optimize the cost associated with transfer trajectories to **libration point** orbits in the Sun-Earth-Moon four body problem, that may include lunar gravity assists. Initially, dynamical systems theory is used to determine invariant manifolds associated with the desired libration point orbit. These manifolds are employed to produce an initial approximation to the transfer trajectory. Specific trajectory requirements such as, transfer injection constraints, inclusion of phasing loops, and targeting of a specified state on the manifold are then incorporated into the design of the transfer trajectory. A two level differential corrections process is used to produce a fully continuous trajectory that satisfies the design constraints, and includes appropriate lunar and solar gravitational models. Based on this methodology, and using the manifold structure from dynamical systems theory, a technique is presented to optimize the cost associated with insertion onto a specified libration point orbit.

GENESIS
halo orbit

Procedure

The procedure to determine a suitable transfer trajectory begins with the selection of a libration point orbit (LPO) that meets the desired mission criteria. Based on this periodic or quasi-periodic orbit, then, a series of stable manifolds are determined that approach the reference orbit asymptotically.[1−3] From dynamical systems theory it is known that the stable and unstable manifolds associated with periodic (and quasi-periodic) solutions form surfaces in the six dimensional phase space (position plus velocity).[1−3] Moreover, these manifolds appear as two dimensional surfaces when projected onto three dimensional configuration space (position only). States that lie on this surface will asymptotically approach the reference orbit, provided that the state matches the entire 7 dimensional state (position, velocity, and time) on the manifold at the specified point.

Along this surface representing the stable manifold, a single trajectory is selected that contains an appropriate close approach to the Earth. This solution serves as the initial approximation to the transfer from the vicinity of the Earth to the libration point orbit. In general, however, the Earth close approach will not satisfy the necessary transfer trajectory injection (TTI) constraints, such as altitude or inclination. Thus, the methodology described in Howell and Wilson[4,5], Wilson[6], and Howell et al.[1] is employed to enforce the desired transfer injection conditions, as well as, to target the desired manifold.

To apply this methodology, a single state is selected along the desired stable manifold to serve as a fixed target point for the end of the transfer. The fixed position and time corresponding to this state on the manifold surface are targeted by a two level differential corrections process to produce the complete transfer trajectory. In order to precisely approach the desired libration point orbit, the final state on the transfer must, in fact, lie on the surface defined by the stable manifold. The position and time requirements can be met by this procedure, however, the velocity at the final state is not constrained in the solution process. Therefore, a maneuver is required to correct any velocity discontinuity between the end of the transfer and the required velocity state on the manifold; this maneuver is called the Libration Orbit Insertion or LOI. Once the state of the vehicle is actually on the manifold surface, it will then approach the libration point orbit; this completes the transfer from Earth to the LPO with, theoretically, no additional maneuvers.

Selection of an Optimal LOI Location

The selection process for the fixed LOI location is somewhat arbitrary, but the target state is generally chosen to produce a reasonable insertion cost onto the desired LPO. It is desired to allow this fixed state to vary in some specified manner in order to determine a more optimal location for the LOI maneuver. An automated procedure is developed to vary the position and time of this final target state, while preserving the manifold solution obtained from dynamical systems theory. Thus, the resulting transfer trajectory will still insert onto the same manifold surface and hence, approach the desired libration point orbit.

Utilizing the previous procedure, the selected final target state on the manifold surface generates a transfer that may or may not correspond to a solution with an acceptable LOI cost. A methodology is sought to allow the "fixed" LOI target state to vary along the two dimensional manifold surface to minimize the required insertion maneuver. Schematically, this is depicted in Figure 1. Initially, the target state $\bar{X}_{act}$ for the transfer lies on the desired manifold surface in position and time, but requires some associated LOI cost to achieve the 7 dimensional manifold state that will approach the libration point orbit. Based on this velocity difference, a change in state $\Delta\bar{X}$ is calculated to reduce the magnitude of the required maneuver. This results in a new final state $\bar{X}_{des}$ that, in all likelihood, does not lie on the required manifold surface. However, if this new final state is projected back onto the manifold surface, another state $\bar{X}_{proj}$ is obtained that does lie on the desired surface, and

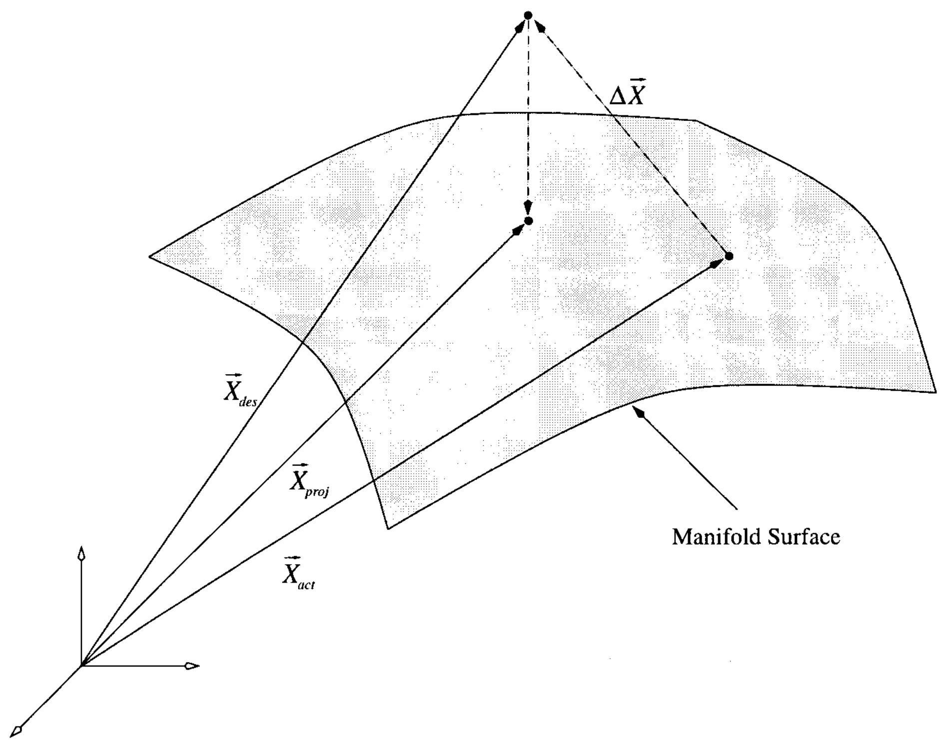

Figure 1: Stylized Representation of Manifold Targeting Procedure

therefore, is an acceptable final target state. A new transfer is determined to this new LOI location that should require a smaller maneuver to insert onto the manifold. This iterative process is repeated until some minimum cost is achieved.

One Dimensional Variations Along the Manifold

As an example of the application of this methodology, consider the variation of the LOI target state along a single manifold trajectory. In this case, the "surface" is, in fact, one dimensional, corresponding to the selected manifold solution. Some LOI target state $\bar{X}_{act}$ along the manifold is selected and the transfer is computed to meet the desired position and time. From this computed solution, changes in the position and time ($\Delta\bar{X}$) of the final state are determined that will reduce the velocity error. This change in state is added to the previous target position and time to produce a new final target state $\bar{X}_{des}$; however, this point no longer lies on the desired manifold surface.

By projecting $\bar{X}_{des}$ onto the one dimensional manifold trajectory, a new final target state $\bar{X}_{proj}$ is determined. This projection is computed by minimizing the distance from the desired target, $\bar{X}_{des}$, to some point on the actual manifold trajectory. Once the new LOI target state is determined from $\bar{X}_{proj}$, a new transfer is computed using the previous solution as an initial guess. A new LOI cost is computed and the process is repeated until some minimum insertion cost is achieved. Note that time is selected as the independent variable along the manifold when projecting the desired end state onto the manifold surface. The time along the manifold is monotonic and provides a one-to-one mapping along the trajectory, i.e., there is only one state associated with each time along the one dimensional manifold. To ensure an adequate resolution for the time variable along the manifold, a 10^{th} order interpolation scheme is used with nodes selected every one day along the numerically integrated path. This proves to be an efficient method to both store and evaluate the manifold states over a given time interval.

Results for 1-D Variations Along a Selected Manifold

As an example of this process, consider the direct transfer from Earth to a Lissajous orbit about the Sun-Earth L_2 point using a single lunar gravity assist. The one dimensional manifold trajectory selected for this analysis is plotted in Figure 2 in a frame centered at the Earth that rotates with the Earth about the Sun, such that the x axis is always directed along the line from the Sun to the Earth. In the figure, the manifold path extends from the lunar orbit to the state at JD 2454560.0, approximately half way through the first revolution along the Lissajous orbit. The square symbols on the plot denote 10 day intervals beginning at JD 2454370.0, just after the lunar encounter. The "nominal" LOI point at JD 2454400.0 is also marked. To isolate the effects of the variations in the LOI target state location on the LOI cost, the transfer injection date is fixed at some value that is within the range identified for the given nominal. The transfer to the nominal LOI state is computed as described earlier, and then the LOI target state variation scheme is applied to this solution.

The results of this procedure are presented in Figure 3 for the direct transfer case (i.e., a transfer with no phasing loops). In the figure, the LOI maneuver cost is plotted as a function of the LOI date for a series of transfer injection dates. (To clarify the figure, the abscissa corresponds to LOI target Julian date minus 2454000.) Each curve in the figure

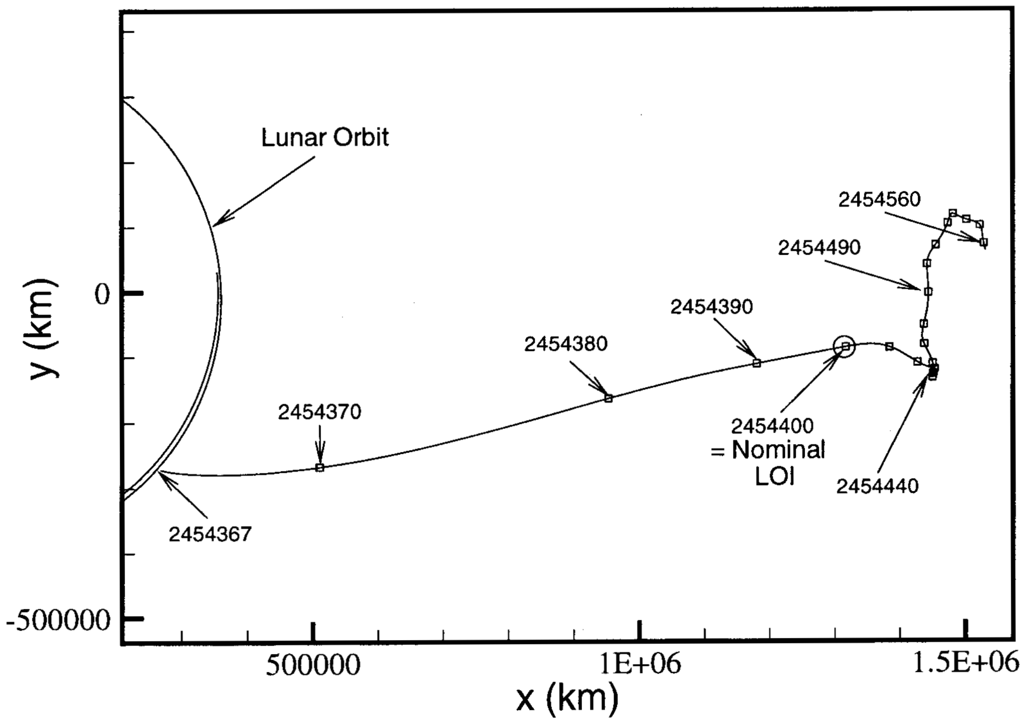

Figure 2: Selected Manifold Trajectory for Earth-to-L_2 Transfer Example

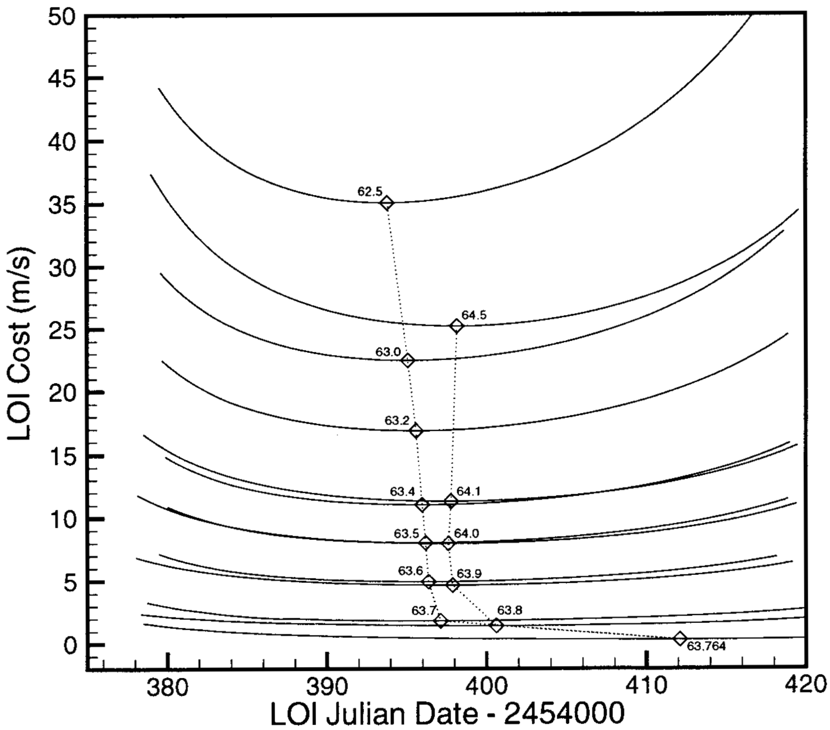

Figure 3: LOI Target Date Variation for Direct Transfer Case

represents the variation in LOI cost for a specified transfer injection date; for example, the curve labeled 62.5 corresponds to solutions with transfer injection on JD (24543)62.5. The minimum LOI cost determined by the LOI target state variation procedure for each given TTI date curve is marked with a diamond. These minimums are connected by a dotted line to signify that a continuum of solutions are possible over the range of TTI dates examined. Note that the overall minimum LOI cost determined by this procedure for the range of dates examined is 0.31 m/s on JD 2454412.058, corresponding to a transfer injection date of JD 2454363.764.

Conclusions

This procedure is highly applicable to a variety of libration point missions, such as the one depicted here using a lunar gravity assist to facilitate the transfer to a Sun-Earth L_2 Lissajous. Similar results are available for this type of transfer that includes multiple phasing loops prior to the lunar encounter. This process is also useful in missions without lunar gravity assists, such as the upcoming GENESIS Discovery mission.[1] Extension of the one dimensional results to the full two dimensional surface will allow an optimal LOI location to be determined on the manifold surface, while maintaining the desired characteristics of the libration point orbit.

References

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